Enrollment No: ____________________ **Exam Seat No:** ____________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Linear Algebra-I
Subject Code: 4SC03LIA1 **Branch: B.Sc. (Mathematics)**
Semester: 3 **Date: 15/03/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Basis and Dimension **(02)**

b) State Cauchy-Schwarz's inequality. **(02)**

c) Define: Sub Space **(01)**

d) Union of two subspaces is subspace.-True or False? **(01)**

e) $\dim(P_4) = \_\_\_\_\_$. **(01)**

f) For a bijective mapping $T: R^5 \to R^5$ then the rank of T is _____. **(01)**

g) $T = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ is the matrix form for shear in the y-direction on R^2.- True or False? **(01)**

h) Find $d(u,v)$ if $u = (u_1, u_2) = (2,5)$; $v = (v_1, v_2) = (5,3)$ & inner product space is $\langle u,v \rangle = 4u_1v_1 - 5u_2v_2$ **(01)**

i) Find the value of k for which the vectors $(2,0,0)$, $(0,4,0)$ and $(0,0,k)$ are linearly dependent. **(01)**

j) Suppose $T: V \to W$ is a linear transformation where $\dim(V) = \dim(W)$. Then which of the following is true? **(01)**

a) T is injective b) T is invertible c) T is surjective d) none of these

k) Let $T: V \to W$ be a linear transformation. What is the rank-nullity formula? **(01)**

a) $\dim(V) = rank(T) - nullity(T)$
b) $\dim(W) = rank(T) + nullity(T)$
c) $\dim(W) = rank(T) - nullity(T)$
d) $\dim(V) = rank(T) + nullity(T)$

l) Define : Angle between two vectors. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let $V = \{(a_1, a_2): a, b \in \boldsymbol{R}\}$ and $F = \boldsymbol{R}$ with the given addition and scalar multiplication of V as follows: **(07)**

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1 + 1, a_2 + b_2 + 1)$$

$$\alpha(a_1, a_2) = (\alpha a_1 + a_1 - 1, \alpha a_2 + a_2 - 1).$$

Show that V is a vector space over $\boldsymbol{R}$.

b) Define: Vector space. **(07)**

Also Check whether the following are subspaces of vector space V.

i) $W = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}; \quad V = R^3$

ii) $W = \{(x, y, 3) \in \boldsymbol{R}^3 : x, y \in \boldsymbol{R}\}; V = \boldsymbol{R}^3$.

iii) $W = \{(x, y, z) \in \boldsymbol{R}^3 : 2x - y + z = 1, x - 3y + z = 0\}; V = \boldsymbol{R}^3$

Q-3 **Attempt all questions** **(14)**

a) Which of the following are linear transformations? **(06)**

i) $T: p_2 \to p_2$, where $T(a_0 + a_1 x + a_2 x^2) = a_0 + a_1(x + 1) + a_2(x + 1)^2$

ii) $T: R^2 \to R^2; \quad T(x, y) = (3x + 2y, 5x - 6y)$

b) Prove that the set $S = \{(1,2,1), (2,1,1), (1,1,2)\}$ is Basis of $\boldsymbol{R}^3$. **(04)**

c) Express $(3, 4, 6)$ as a linear combination of $\{v_1, v_2, v_3\}$, Where **(04)**

$v_1 = (1, -2, 2), v_2 = (0, 3, 4), v_3 = (1, 2, -1)$.

Q-4 **Attempt all questions** **(14)**

a) Consider the basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = (1, 1), v_2 = (1, 0)$ and and let $T: R^2 \to R^2$ be the linear transformation such that $T(v_1) = (1, -2)$ and $T(v_2) = (-4, 1)$. Find $T(x, y)$ and $T(5, -3)$. **(05)**

b) If $T: R^2 \to R^2$ defined by $T(x, y) = (2x - y, 8x + 4y)$ then find the range of T, rank of T, Ker (T) and nullity of T. **(05)**

c) Find the domain and co-domain of $T_2 {}^\circ T_1$ and find $(T_2 {}^\circ T_1)(x, y)$. **(04)**

a. $T_1(x, y) = (x - 3y, 0)$ and $T_2(x, y) = (4x - 5y, 3x - 6y)$

b. $T_1(x, y) = (2x, -3y, x + y)$ and $T_2(x, y) = (x - y, y + z)$

Q-5 **Attempt all questions** **(14)**

a) Let $T: R^3 \to R^3$ be the L.T. defined by $T(x, y, z) = (x + 2y + z, 2x - y, 2y + z)$. **(07)**

Find the matrix representation of T with respect to basis

(i) S_1

(ii) S_1 and S_2

(iii) S_2 and S_1,

where $S_1 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $S_2 = \{(1, 0, 1), (0, 1, 1), (0, 0, 1)\}$ for R^3.

b) Prove that linear transformation $T: R^3 \to R^3$ is isomorphism. **(04)**

Where $T(x, y, z) = (x + 3y, y, 2x + z)$

c) Explain: Reflection operators **(03)**

Q-6 **Attempt all questions** **(14)**

a) Which of the following set S of vectors/polynomials in vector space V are linearly dependent or linearly independent? **(06)**

i) $S=\{(4,-1,2),(-4,10,2),(4,0,1)\}$; $V=R^3$

ii) $S=\{2+x+x^2, x+2x^2, 2+2x+3x^2\}$; $V=P_2$

b) Let $T:R^3\to R^3$ be linear transformation defined by $T(x_1,x_2,x_3)=(x_1-x_2+x_3, 2x_2-x_3, 2x_1+3x_2)$ determine whether T is one-one. If so find $T^{-1}(x_1,x_2,x_3)$. **(06)**

c) Check whether the set $V=\{(x,e^x): x>0\}$ is a vector space or not with the given operation: $(x,e^x)+(y,e^y)=(x+y,e^{x+y})$ and $(x,e^x)=(\alpha x,e^{\alpha x})$. **(02)**

Q-7 **Attempt all questions** **(14)**

a) Let S be a finite set of vectors in a vector space V over a field F. The set of all linear combinations of the vectors in S forms a smallest subspace of V containing S. **(06)**

b) If V and W are vector spaces over a field F and $T:V\to W$ a linear transformation then $Ker(T)$ is a subspace of V. **(04)**

c) Find the space generated by $v_1=(1,3,0)$, $v_2=(2,1,-2)$. Examine if $v_3=(-1,2,3)$, $v_4=(4,7,-2)$ are in the space. **(04)**

Q-8 **Attempt all questions** **(14)**

a) Find the cosine angle between given vectors and also verify Cauchy-Schwarz inequality for $u=(1,0,1,0)$ & $v=(-3,-3,-3,-3)$. **(05)**

b) Find $\langle f,g\rangle$, $\|f\|$ and $\|g\|$, if $f(x)=3x-5$ and $g(x)=x^2+1$ and the inner product is defined by $\langle f,g\rangle=\int_0^1 f(x)g(x)\,dx$. **(05)**

c) Prove that $\langle u,v\rangle=2u_1v_1+u_2v_2+4u_3v_3$ is an inner product space on R^3. **(04)**